# Modeling Exponential Growth and Decay

In real-world applications, we need to model the behavior of a function. In mathematical modeling, we choose a familiar general function with properties that suggest that it will model the real-world phenomenon we wish to analyze.

For rapid growth and decay, we may choose to use the exponential function

where is equal to the value at time zero, is Euler’s constant, and is a constant that determines the rate (percentage) of growth/decay.

## Exponential Growth Models

We may use the exponential growth function in applications involving **doubling time**, the time it takes for a quantity to double. Such phenomena as wildlife populations, financial investments, biological samples, and natural resources may exhibit growth based on a doubling time.

For rapid growth, we can use the exponential function

where is equal to the value at time zero, is Euler’s constant, and is a **positive** constant that determines the rate (percentage) of growth.

Example: A population of bacteria doubles every hour. If the culture started with 10 bacteria, graph the population as a function of time.

## Exponential Decay Models

If a quantity is falling rapidly toward zero, without ever reaching zero, then we should probably choose the exponential decay model. We may use the exponential decay model when we are calculating half-life, or the time it takes for a substance to exponentially decay to half of its original quantity. Another important application of decay is radiocarbon dating, which is used to calculate the approximate date a plant or animal died.

For rapid decay, we can use the exponential function

where is equal to the value at time zero, is Euler’s constant, and is a **negative** constant that determines the rate (percentage) of decay.

Examples:

1. The half-life of carbon- is years. Express the amount of carbon- remaining as a function of time, .
2. A bone fragment is found that contains of its original carbon-. To the nearest year, how old is the bone?

# Choosing an Appropriate Model for Data

Now that we have discussed various mathematical models, we need to learn how to choose the appropriate model for the raw data we have. Many factors influence the choice of a mathematical model, among which are experience, scientific laws, and patterns in the data itself. Not all data can be described by elementary functions. Sometimes, a function is chosen that approximates the data over a given interval.

Three kinds of functions that are often useful in mathematical models are **linear functions**, **exponential functions**, and **logarithmic functions**. If the data lies on a straight line, or seems to lie approximately along a straight line, a linear model may be best. If the data is non-linear, we often consider an exponential or logarithmic model.

Example: Does a linear, exponential, or logarithmic model best fit the values in the table below? Find the model and use a graph to check your choice.

|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | 1.386 | 2.197 | 2.773 | 3.219 | 3.584 | 3.892 | 4.159 | 4.394 |